ECE 333 – Renewable Energy Systems

2. Power System Basics – AC Analysis

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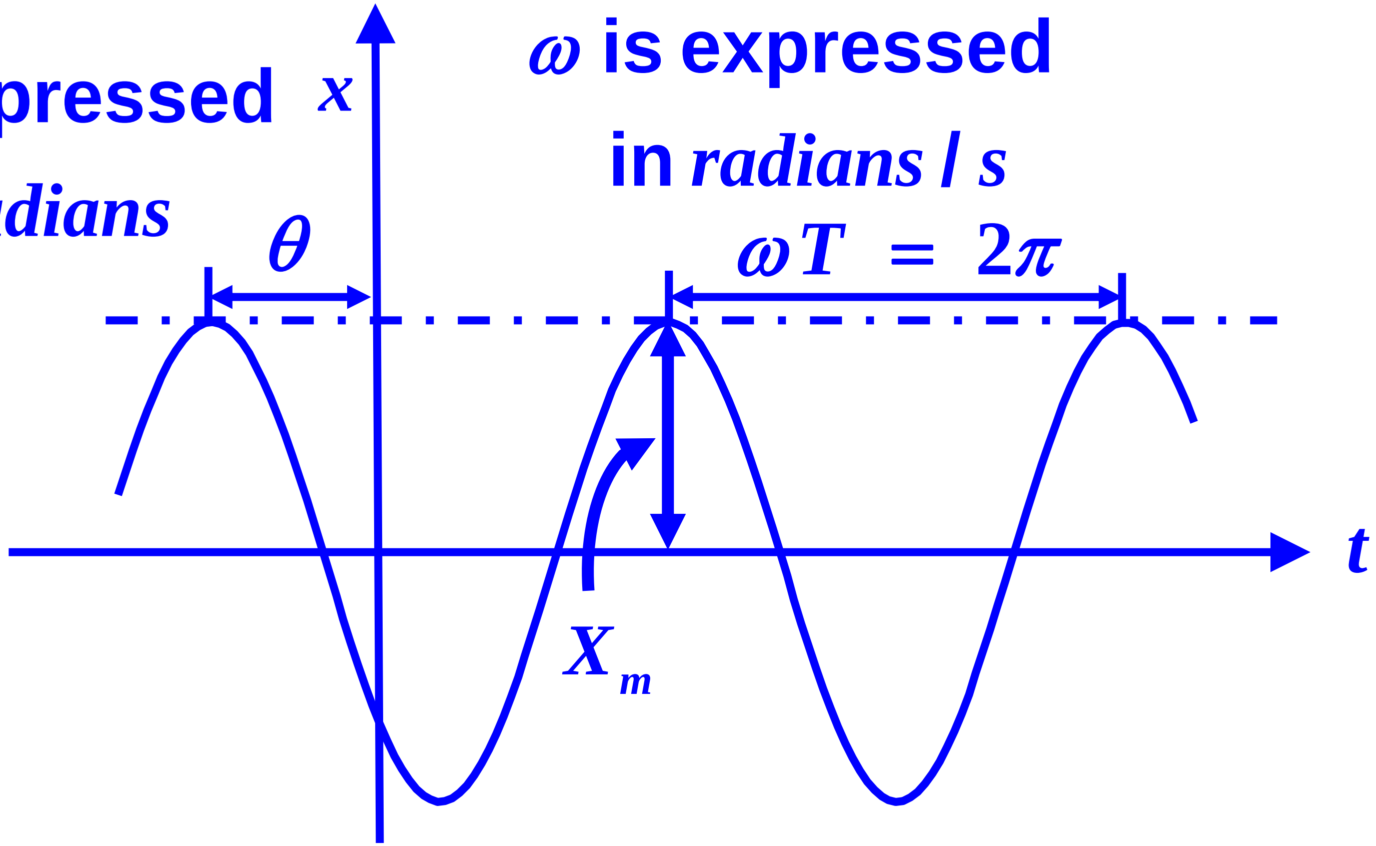
SINUSOIDAL VARIABLES

- In AC circuits, the basic current and voltage variables are considered to be sinusoidal functions
- A sinusoidal function is

$$x(\underset{\substack{\uparrow\\ \text{time}}}{t}) = \underset{\substack{\uparrow\\ \text{magnitude}\\ \text{or}\\ \text{amplitude}}}{X_m} \cos(\underset{\substack{\uparrow\\ \text{angular}\\ \text{frequency}}}{\omega} t + \underset{\substack{\uparrow\\ \text{phase}\\ \text{angle}}}{\theta})$$

SINUSOIDAL VARIABLES

- The argument of the sinusoidal function is radians and so:



θ is expressed in *radians*

ω is expressed in *radians / s*

ANGULAR FREQUENCY ω

- We can also express the argument of the sinusoidal function in terms of the frequency f given in *Hz* or cycles per second with

$$\omega = 2\,\pi\,f$$

radians / cycle

Hz

radians/s

ANGULAR FREQUENCY ω

- **The periodic sinusoidal function has a period of T s, where**

$$\textit{s/cycle} \longrightarrow T = \frac{1}{f} \longleftarrow \textit{cycles/s}$$

that is each cycle (or period) takes $T\,s$

- **We may express $x(t)$ therefore as**

$$x(t) = X_m \cos(\omega t + \theta) = X_m \cos(2\pi f t + \theta)$$

$$= X_m \cos\left(\frac{2\pi}{T} t + \theta\right)$$

AC SYSTEM

- ☐ The current in the AC system is specified by

$$i(t) = I_m \ cos(\omega t + \theta_i)$$

- ☐ The use of the cosine function is rather *arbitrary* since for arbitrary angle ϕ

$$sin(\phi) = cos\left(\frac{\pi}{2} - \phi\right)$$

or equivalently

$$cos(\phi) = sin\left(\frac{\pi}{2} - \phi\right)$$

AC SYSTEM

- ☐ The voltage is also sinusoidal

$$v(t) = V_m \cos(\omega t + \theta_v)$$

- ☐ The power is

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_i)\cos(\omega t + \theta_v)$$

- ☐ Recall that

$$\cos\phi \cos\xi = \frac{1}{2}\left[\cos(\phi + \xi) + \cos(\phi - \xi)\right]$$

POWER EXPRESSION FOR NETWORK

- We are interested in evaluating the average value of $p(t)$: since we are focusing on two periodic functions each with the identical period T, the average value is the same as the average value over a period

AC SYSTEM

- ☐ **Therefore**

$$p_{avg} = \frac{1}{T}\int_{0}^{T} p(t)\,dt$$

$$= \frac{V_m I_m}{2T}\int_{0}^{T}\left[\cos\left(2\omega t + \theta_i + \theta_v\right) + \cos\left(\theta_i - \theta_v\right)\right]dt$$

$$= \frac{V_m I_m}{2T}\cos\left(\underbrace{\theta_i - \theta_v}_{\theta}\right)\cdot T$$

AC SYSTEM

where we use the fact that the average value of a sinusoid is 0, as the positive and negative areas cancel out

- ☐ Therefore

$$p_{avg} = \frac{1}{2} V_m I_m \cos\theta$$

EFFECTIVE VALUE

- ☐ **The effective value of a periodic variable is the square root of the average of the squared value of the variable**

- ☐ **For current**

$$I \triangleq \left[\frac{1}{T} \int_{0}^{T} i^{2}(t)\, dt \right]^{\frac{1}{2}}$$

EFFECTIVE VALUE

- ☐ **We next evaluate I**

$$I = \left[\frac{1}{T} \int_{0}^{T} I_m^2 \underbrace{\cos^2\left(\omega t + \theta_i\right) dt}_{\displaystyle \downarrow} \right]^{\frac{1}{2}}$$

$$\underbrace{\frac{1}{2}\left[\cos 2\left(\omega t + \theta_i\right) + \cos\left(0\right)\right]}_{\displaystyle \downarrow}$$

$$\frac{1}{2}\left[1 + \cos 2\left(\omega t + \theta_i\right)\right]$$

$$I = \frac{I_m}{\sqrt{2}}$$

EFFECTIVE VALUE

- ☐ **The value I is referred to as the *r.m.s.* value**
- ☐ **The *r.m.s.* value of a sinusoid equals its amplitude divided by $\sqrt{2}$**
- ☐ **The 240-V 60-Hz voltage at which electricity is supplied to a dryer is understood to mean that**

$$V = 240 \; V$$

and so

$$V_m = 240\sqrt{2} = 339.41 V$$

AC SYSTEM

$$V_m = 240\sqrt{2} = 339.41V$$

with the angular frequency

$$\omega = 2\pi \cdot 60 = 377 radians/s$$

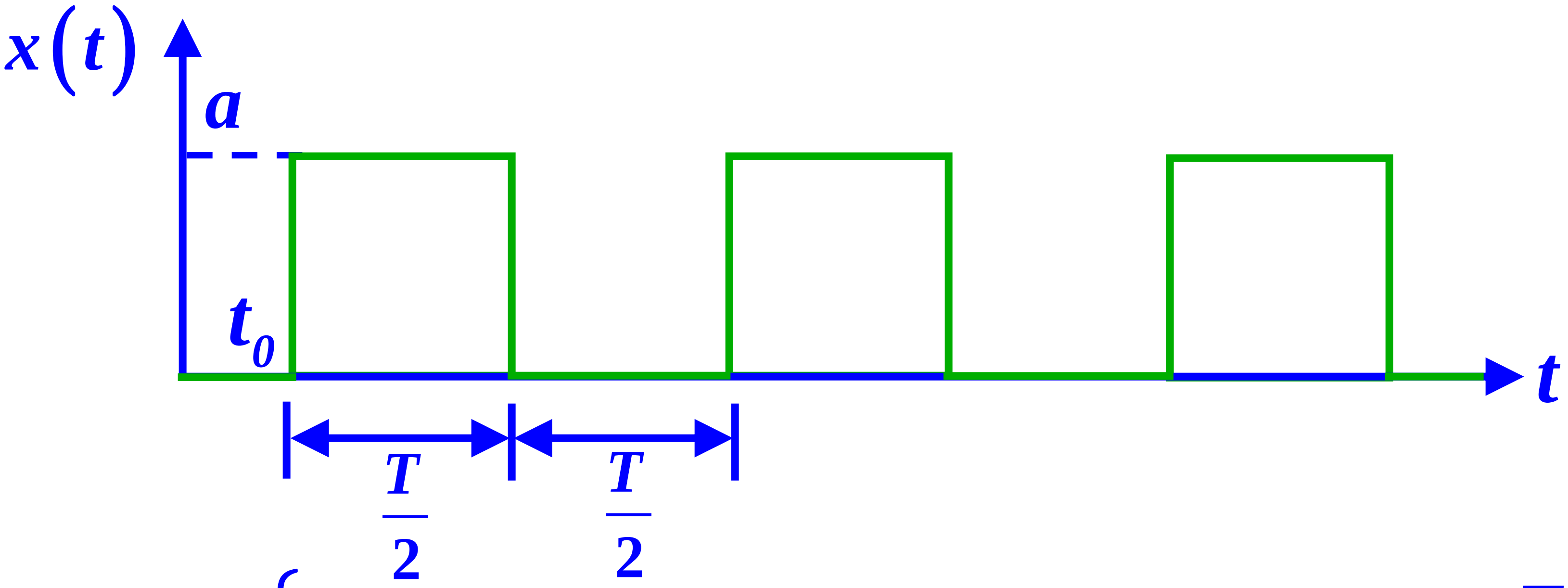
and so we have the sinusoid

$$v(t) = 339.41 \, cos(377t + \theta_v)$$

- ☐ We henceforth adopt the convention of treating incoming voltage as having $\theta_v = 0$ and we measure all other variables with respect to the reference voltage

r.m.s. VALUE OF A SQUARE WAVE

- We consider the *square wave*



$$x(t) = \begin{cases} a & t_0 + (n-1)T \le t \le t_0 + (2n-1)\frac{T}{2} \\ & \qquad\qquad\qquad n = 1, 2, \ldots \\ 0 & t_0 + (2n-1)\frac{T}{2} < t < t_0 + nT \end{cases}$$

r.m.s. VALUE OF A SQUARE WAVE

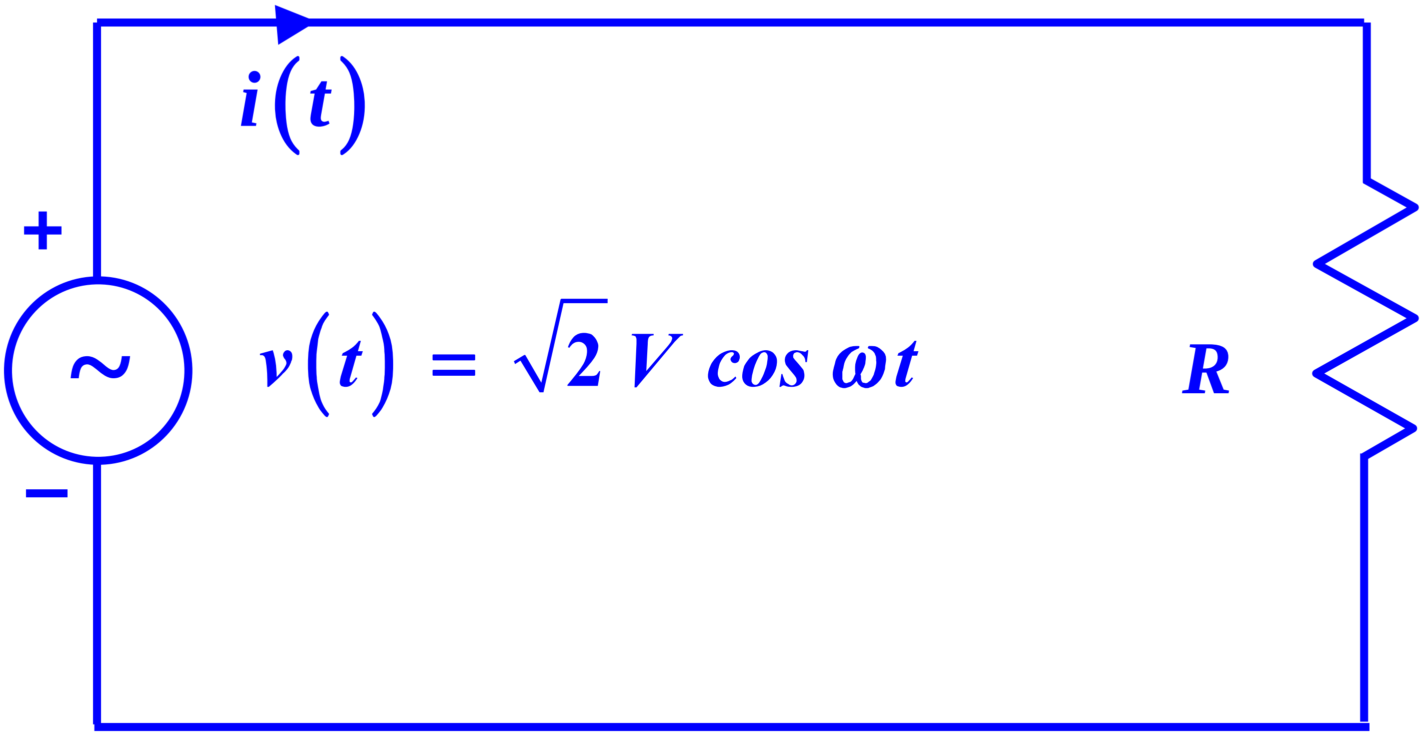
- ❑ We compute the *r.m.s.* value of $x(t)$ by evaluating the average value over a cycle

$$X = \left\{ \frac{1}{T} \int_{t_0}^{t_0 + T} \left[x(t) \right]^2 dt \right\}^{\frac{1}{2}} = \left\{ \frac{1}{T} \int_{t_0}^{t_0 + \frac{T}{2}} a^2 \, dt \right\}^{\frac{1}{2}}$$

$$= \left[\frac{a^2}{T} \cdot \frac{T}{2} \right]^{\frac{1}{2}}$$

$$= \frac{a}{\sqrt{2}}$$

IDEAL RESISTOR IN AC NETWORKS



IDEAL RESISTOR IN AC NETWORKS

- ❑ We analyze the behavior of an ideal resistor in a circuit with a sinusoidal voltage source

$$v(t) = V_m \cos \omega t$$

or in the terms of the *r.m.s.* voltage *V*

$$v(t) = \sqrt{2} V \cos \omega t$$

- ❑ Now,

$$i(t) = \frac{v(t)}{R} = \sqrt{2} \frac{V}{R} \cos \omega t$$

IDEAL RESISTOR IN AC NETWORKS

is the current through the resistor with *r.m.s.* value

$$I = \frac{V}{R}$$

- ☐ Since there is a 0 angle phase difference between the $v(t)$ and $i(t)$ sinusoids, we say that the two sinusoids are *in phase* with each other

IDEAL RESISTOR IN AC NETWORKS

- ☐ The evaluation of the average power is

$$p_{avg} = VI \cos\underbrace{\left(\theta_v - \theta_i\right)}_{0} = VI = \frac{V^2}{R} = I^2R$$

- ☐ In AC networks, power is always interpreted as average power and so we drop the *avg* subscript and write

$$P = V\,I = I^2R = \frac{V^2}{R}$$

and P represents the average power

EXAMPLE: CUISINART TOASTER

- The two-slot Cuisinart toaster uses 1,500 *W* of power when plugged into a 120-*V* socket at 60 *Hz*; the appliance is modeled as a simple resistor
- We compute from

$$P = \frac{V^2}{R}$$

the value of the resistance

$$R = \frac{V^2}{P} = \frac{120 \cdot 120}{1,500} = \frac{14,400}{1,500} = 9.6\ \Omega$$

EXAMPLE: CUISINART TOASTER

- The current is

$$I = \frac{V}{R} = \frac{120}{9.6} = 12.5\ A$$

- Now suppose there is a voltage spike of $125\,V$ and so the dissipated power becomes

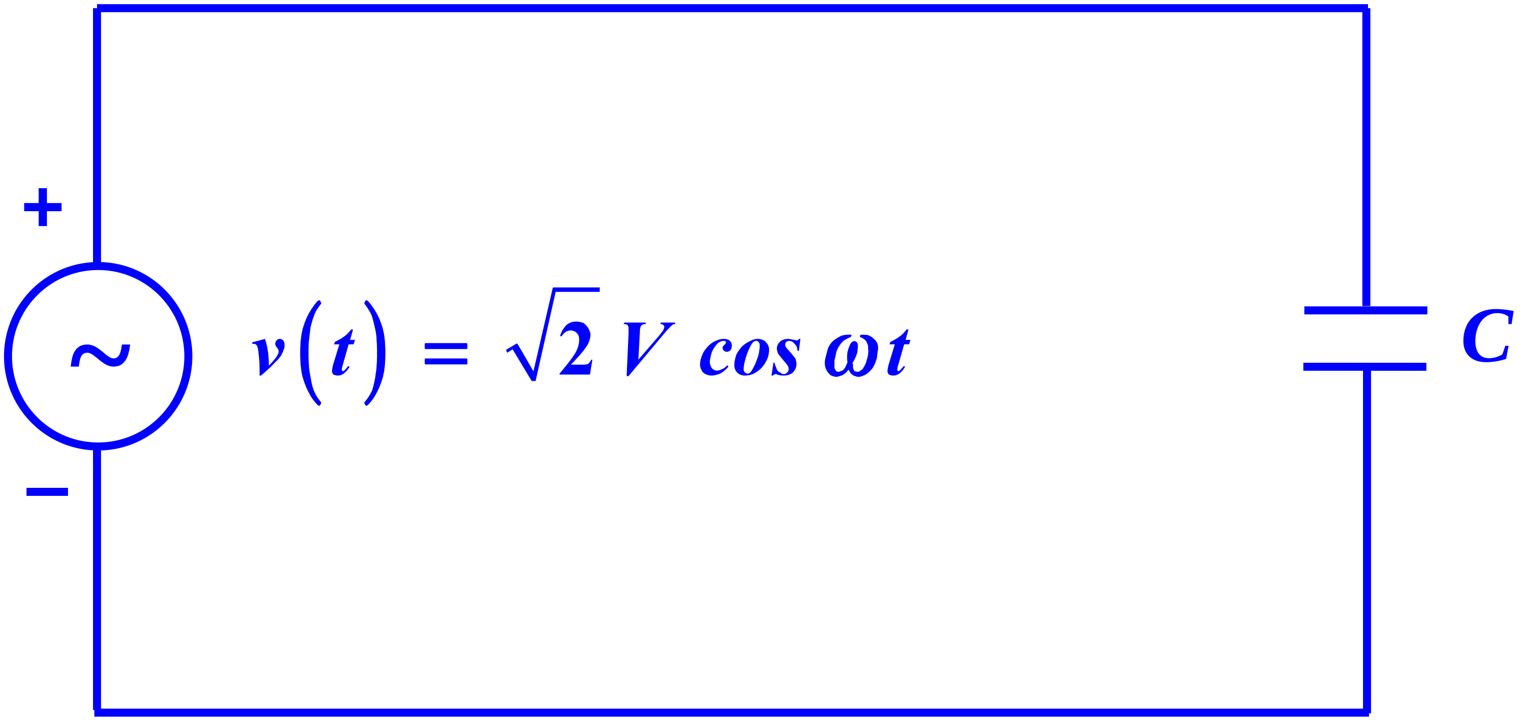
$$P = \frac{V^{2}}{R} = \frac{125 \cdot 125}{9.6} = 1627.6\ W$$

representing an increase of $127.6\,W$ in the toaster consumption – an $8.5\,\%$ increase

IDEALIZED CAPACITOR IN AC NETWORKS

- **Recall the *equation of motion* for a capacitor**

$$i(t) = C\,\frac{dv}{dt}$$



IDEALIZED CAPACITOR IN AC NETWORKS

- ☐ **For a sinusoidal voltage in an AC network**

$$i(t) = C\frac{d}{dt}\left[\sqrt{2}V\cos\omega t\right] = -\omega C\sqrt{2}V\sin\omega t$$

- ☐ **We use the identity**

$$\sin\phi = \cos\left(\frac{\pi}{2} - \phi\right) = -\cos\left(\frac{\pi}{2} - \phi - \pi\right) = -\cos\left(\phi + \frac{\pi}{2}\right)$$

- ☐ **Thus**

$$i(t) = \omega C\sqrt{2}V\cos\left(\omega t + \frac{\pi}{2}\right)$$

IDEALIZED CAPACITOR IN AC NETWORKS

- Therefore, the voltage across the capacitor and the current through it are
  - same frequency sinusoids
  - there is a $\frac{\pi}{2}$ *radians* shift between the two waveforms
  - the current *leads* the voltage by $\frac{\pi}{2}$ and so

IDEALIZED CAPACITOR IN AC NETWORKS

- ☐ **Let**

$$I = \omega C V$$

and so

$$i(t) = \sqrt{2} I \cos\left(\omega t + \frac{\pi}{2}\right)$$

IDEALIZED CAPACITOR IN AC NETWORKS

- ☐ **We summarize**

$$V = \left(\frac{1}{\omega C} \right) I \longleftarrow$$

AC version of *Ohm's Law* for capacitors

- ☐ **The power dissipated by the capacitor is**

$$p(t) = v(t)\, i(t) = \sqrt{2} V \cos \omega t \, \sqrt{2}\, I \cos\left(\omega t + \frac{\pi}{2} \right)$$

and this simplifies to

IDEALIZED CAPACITOR IN AC NETWORKS

$$p\left(t\right) = 2V\,I \cdot \frac{1}{2}\left[cos\left(2\omega\,t + \frac{\pi}{2} \right) \right] + \underbrace{cos\left(-\frac{\pi}{2} \right)}_{0}$$

$$= V\,I\;cos\left(2\omega\,t + \frac{\pi}{2} \right)$$

- ☐ **Since the average value of a sinusoid is *0***

$$p_{avg} = 0$$

and so for a capacitor

$$P = 0$$

CAPACITOR EXAMPLE

☐ We consider the current through a $200\ \mu F$ capacitor supplied by a $120\text{-}V$ $60\text{-}Hz$ source

☐ The voltage is given by

$$v(t) = \sqrt{2}\, 120 \cos \omega t$$

and the current is therefore

$$i(t) = \sqrt{2}\ I \cos\left(\omega t + \frac{\pi}{2}\right)$$

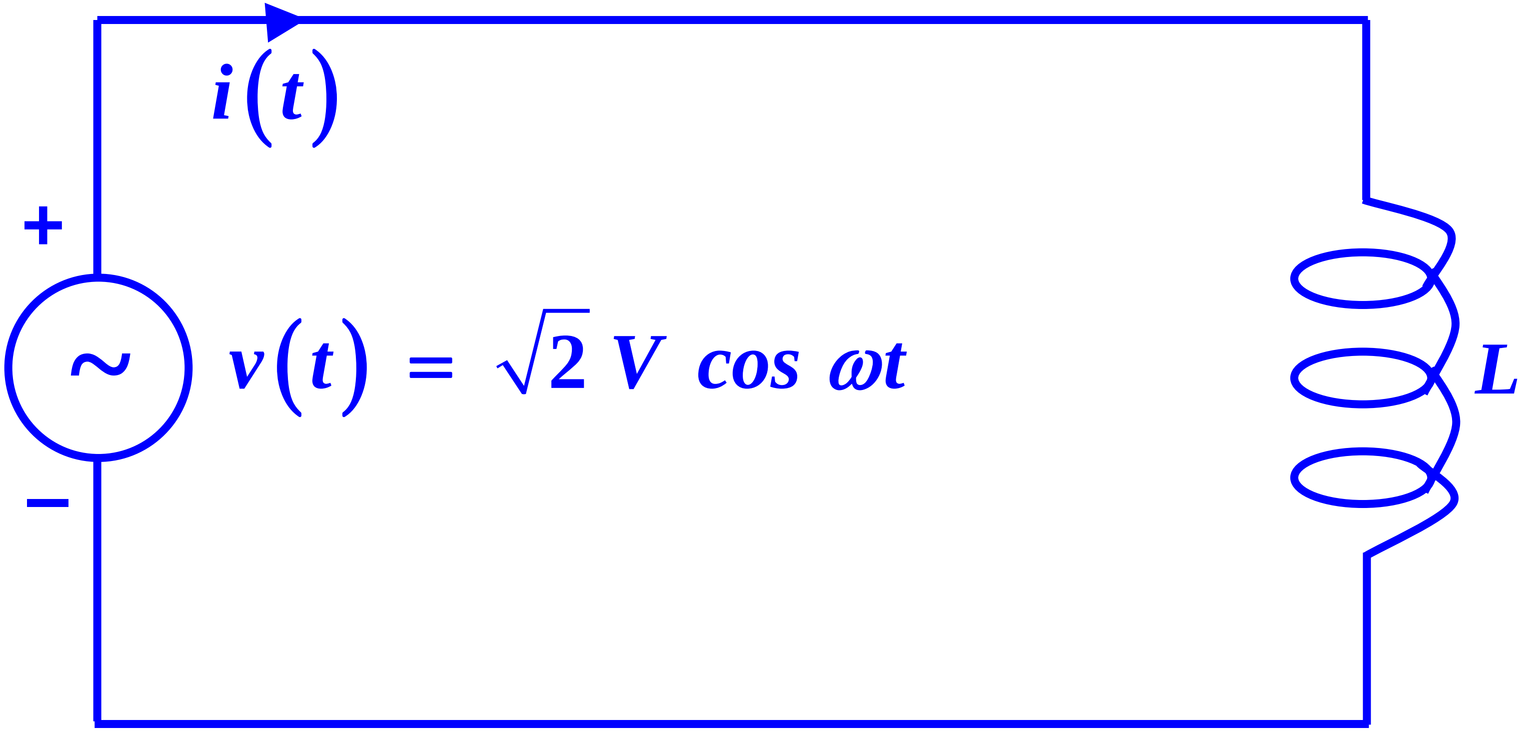
with

$$I = \sqrt{2}\,(2\pi 60)(120)\left(200 \cdot 10^{-6}\right) = 9.048\ A$$

IDEALIZED INDUCTOR IN AC NETWORKS

☐ **Recall the equation of motion for an inductor**

$$v(t) = L\frac{di}{dt}$$



IDEALIZED INDUCTOR IN AC NETWORKS

and so

$$i(t) = \frac{1}{L}\int_0^t v(\xi)\,d\xi$$

- ☐ **For the sinusoidal voltage**

$$v(t) = \sqrt{2}\,V\cos\omega t$$

we have

$$i(t) = \frac{1}{L}\int_0^t \sqrt{2}\,V\,\cos\omega\xi\,d\xi = \frac{\sqrt{2}\,V}{\omega L}\sin\omega t$$

- ☐ **We use the identity**

$$\sin\phi = \cos\left(\phi - \frac{\pi}{2}\right)$$

IDEALIZED INDUCTOR IN AC NETWORKS

- Thus

$$i(t) = \sqrt{2}\,\frac{V}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

- Therefore, the voltage across the inductor and the current through it are
  - same frequency sinusoids
  - there is a $\frac{\pi}{2}$ *radians* shift between the two waveforms

IDEALIZED INDUCTOR IN AC NETWORKS

  - the current *lags* behind the voltage by $\frac{\pi}{2}$

- Let

$$I = \frac{1}{\omega L} V$$

and so

$$i(t) = \sqrt{2}\, I \cos\left(\omega t - \frac{\pi}{2} \right)$$

- We summarize:

$$V = \omega L I$$

AC version of *Ohm's Law* for inductors

IDEALIZED INDUCTOR IN AC NETWORKS

- ❑ The power dissipated by the inductor is

$$p(t) = v(t)\, i(t) = \sqrt{2}V \cos \omega t \, \sqrt{2}\, I \cos\left(\omega t - \frac{\pi}{2}\right)$$

and this simplifies to

$$p(t) = 2VI \cdot \frac{1}{2}\left[\cos\left(2\omega t - \frac{\pi}{2}\right) + \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}\right]$$

$$= VI \cos\left(2\omega t - \frac{\pi}{2}\right)$$

IDEALIZED INDUCTOR IN AC NETWORKS

- Clearly

$$p_{avg} = 0$$

and so

$$P = 0$$

- Neither capacitors nor inductors incur the consumption of real power

POWER FACTOR

- We may generalize the expressions for resistors capacitors and inductors for a sinusoidal

$$v(t) = \sqrt{2}\,V \cos \omega t$$

and a current

$$i(t) = \sqrt{2}\,I \cos(\omega t + \theta)$$

- Now, we have shown that

$$\theta = \begin{cases} 0 & \text{for a resistor} \\ \dfrac{\pi}{2} & \text{for a capacitor} \\ -\dfrac{\pi}{2} & \text{for an inductor} \end{cases}$$

POWER FACTOR

but for a network with an arbitrary combination of R, L and C components, θ is unknown

- ☐ We also showed earlier that the average value of power is

$$p_{avg} = V\, I\, cos\left(\theta\right) \qquad (*)$$

for

$$\theta = \theta_v - \theta_i$$

- ☐ Power engineers define the quantity $cos\,\theta$ as the *power factor*

$$p\,f \triangleq cos\theta$$

POWER FACTOR

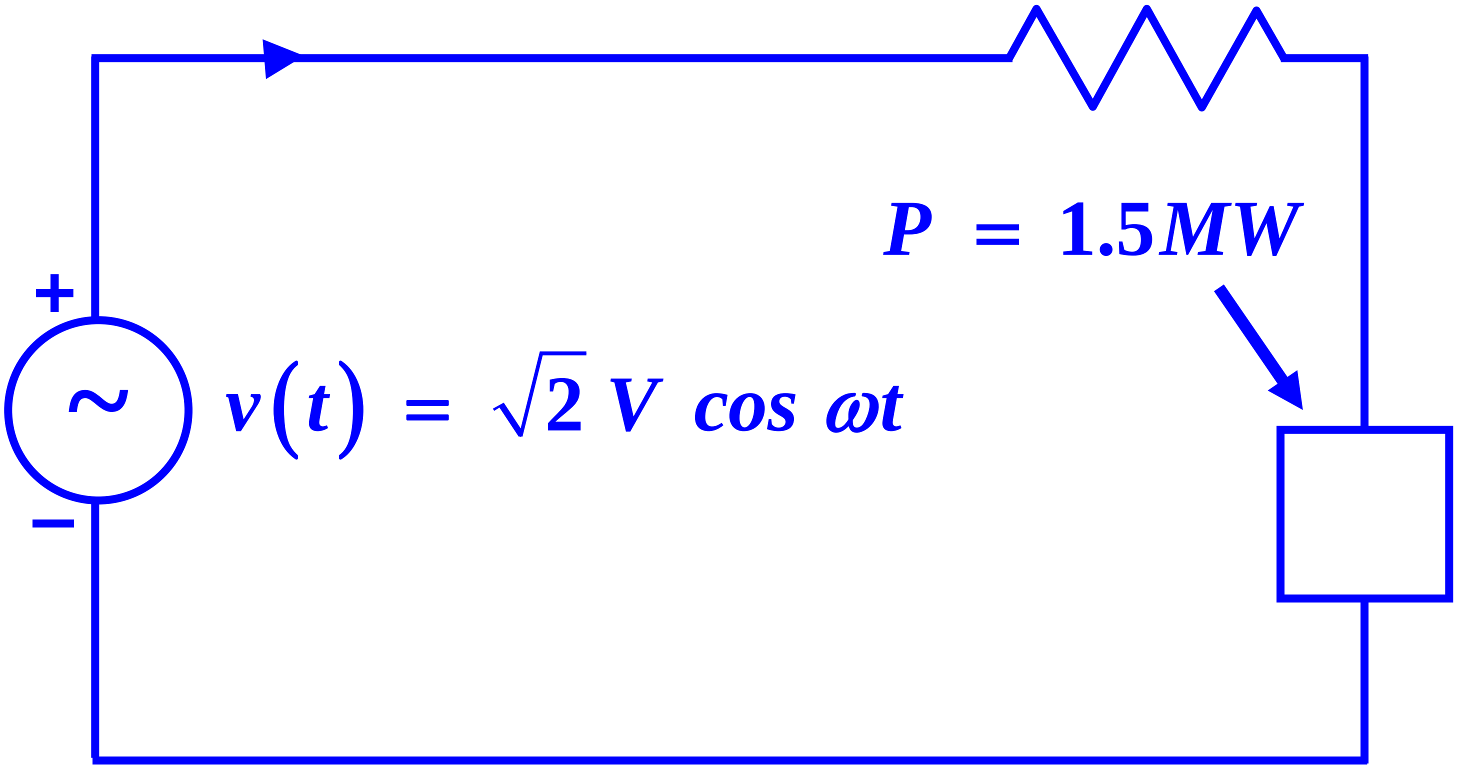
- ☐ The expression in $(*)$ is general and may apply to any circuit or circuit element, any combination of R, L and C elements and more importantly any component with sinusoidal voltage and current
- ☐ The interpretation of pf is the fraction that the real power represents of the total apparent power used by a particular component or system

EXAMPLE ON pf

- ☐ A small industrial customer is supplied by a $24\text{-}kV$ $60\text{-}Hz$ source to run a $1.5\text{-}MW$ real power load through a line with resistance R
- ☐ We compute the ratio of the real power line losses on the feeder line under two different pf values

$$S\Big|_{pf_1} = 0.5 \quad \text{and} \quad S\Big|_{pf_2} = \frac{\sqrt{3}}{2}$$

EXAMPLE ON pf



- ☐ Basic assumption: the voltage drop through R is negligibly small

EXAMPLE ON pf

- Since

$$P = V\,I\,\cos\theta = 1.5\,MW$$

the *r.m.s.* value of the feeder current we compute

under pf_1

$$I_1 = \frac{1.5\,MW}{\frac{1}{2}\left(24\,kV\right)}$$

and also under pf_2

$$\frac{MW}{kV} = kA$$

$$I_2 = \frac{1.5\,MW}{\frac{\sqrt{3}}{2}\left(24\,kV\right)}$$

EXAMPLE ON $p f$

- ☐ The ratio of the losses is therefore

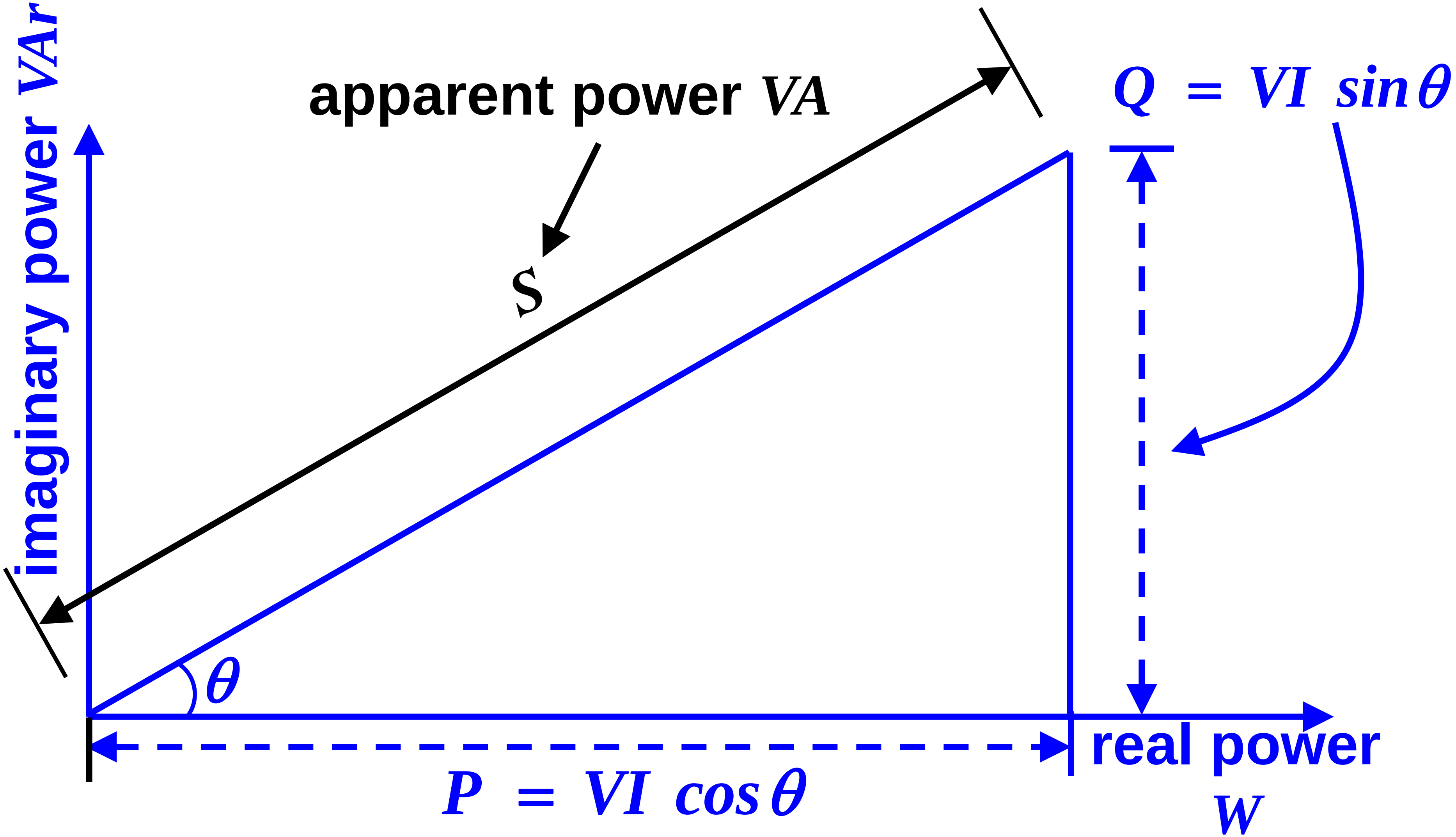
$$\frac{I_1^2 R}{I_2^2 R} = \left(\frac{I_1}{I_2} \right)^2 = \left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \right)^2 = \left(\sqrt{3} \right)^2 = 3$$

- ☐ There are 3 times higher losses under the poor value $p f_1$ than the better value $p f_2$

THE POWER TRIANGLE

- ❑ There is an important relationship between the apparent power S, the real power P and the reactive power Q ; we represent this relationship by the so-called *power triangle* in the complex plane
- ❑ The power triangle is drawn as follows

THE POWER TRIANGLE


apparent power *VA*

S

$Q = VI \sin\theta$

$P = VI \cos\theta$

THE POWER TRIANGLE

$\theta > 0$ current *leads* voltage

$\theta < 0$ current *lags* voltage

$$S = VI$$

$P = S \cos\theta$ ← **real power**

$Q = S \sin\theta$ ← **reactive power**

$S^2 = P^2 + Q^2$ ← **apparent power**

THE POWER TRIANGLE

- For an arbitrary consumptive load

$$P \geq 0$$

but

$Q > 0$ for an inductive load

$Q < 0$ for a capacitive load

- The actual power consumed by a load is the rate at which work is done and is measured in W
- The reactive power is incapable of doing work and its average is always 0 for either a capacitive or an inductive element

THE POWER TRIANGLE

- Power suppliers, typically, charge for P consumption but are impacted also by Q since the larger the Q the larger the line losses; in some cases charges are imposed on the basis of S or take into account the pf
- The presence of electric motors, which constitutes highly inductive loads, leads to increased losses on transmission lines

EXAMPLE: POWER TRIANGLE

- ☐ We consider a $250\text{-}V$ induction motor that draws $20\,A$ of current to generate $4.33\,kW$ of real power delivered to its shaft
- ☐ We draw the power triangle using

$$S = VI = 250 \cdot 20 = 5000\,VA = 5\,kVA$$

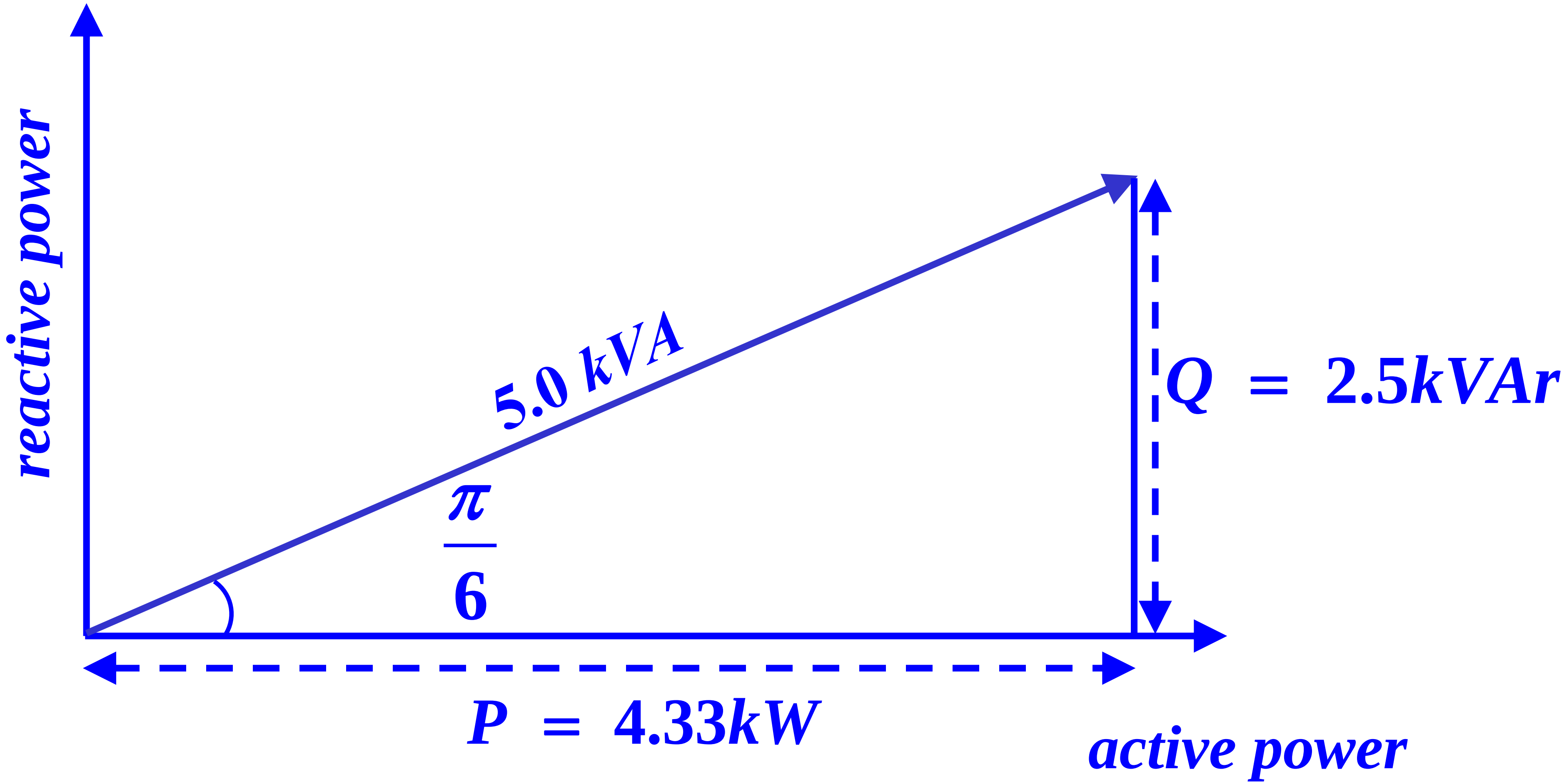
$$P = 4.33\,kW$$

$$\cos\theta = \frac{P}{S} = \frac{4.33}{5} = 0.866$$

$$\theta = \cos^{-1}\left(0.86\right) = \frac{\pi}{6}$$

$$Q = S\sin\theta = 2.5\,kVAr$$

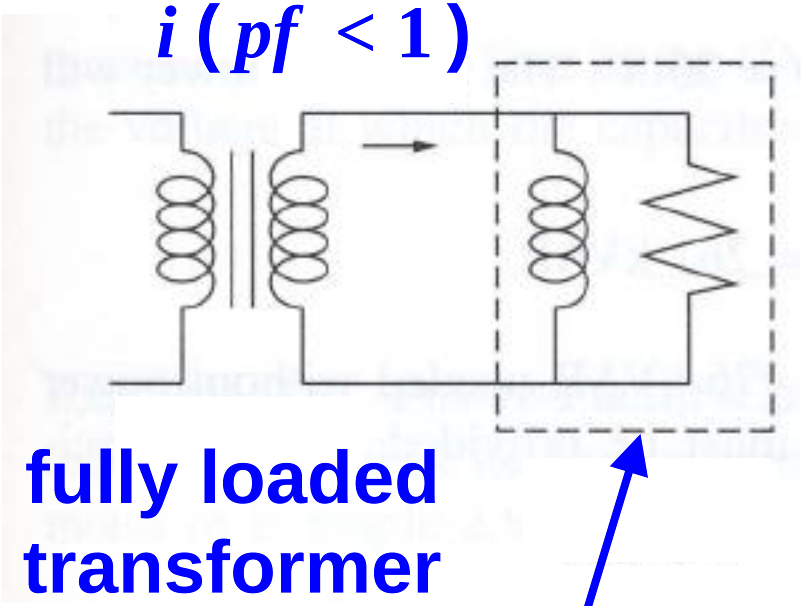
EXAMPLE: POWER TRIANGLE



POWER FACTOR CORRECTION

- The smaller the pf the worse the utilization of power is; the ideal is to get as near as possible to the perfect pf *of* 1.0
- Sometimes it is desirable or necessary to use capacitors to correct the pf to offset the $VArs$ of the inductive elements
- A pf corrective action can lead to increased real power delivery to the loads

EXAMPLE: POWER FACTOR CORRECTION



$i\,(pf < 1)$

fully loaded transformer

load: lagging

pf

EXAMPLE: POWER FACTOR CORRECTION

- ☐ A transformer is operating close to its kVA rating and is used to deliver $600\ kVA$ at a $0.75\ pf$
- ☐ There is a $20\ \%$ forecasted growth in the real power demand
- ☐ This growth needs to be accommodated without investing in a new transformer by installing capacitors for pf correction

EXAMPLE: POWER FACTOR CORRECTION

- ☐ The existing situation is characterized by

$$p\,f \;=\; 0.75 \;=\; \cos\theta$$

$$\theta \;=\; \cos^{-1}\left(0.75\right) \;=\; 0.72\ radians$$

$$P \;=\; 600\cdot 0.75 \;=\; 450\ kW$$

$$Q \;=\; 600\cdot 0.66 \;=\; 397\ kVAr$$

- ☐ The forecasted situation

$$P_{new} \;=\; 450\left(1.2\right) \;=\; 540\ kW$$

$$p\,f_{new} \;=\; \frac{540}{600} \;=\; 0.9$$

EXAMPLE: POWER FACTOR CORRECTION

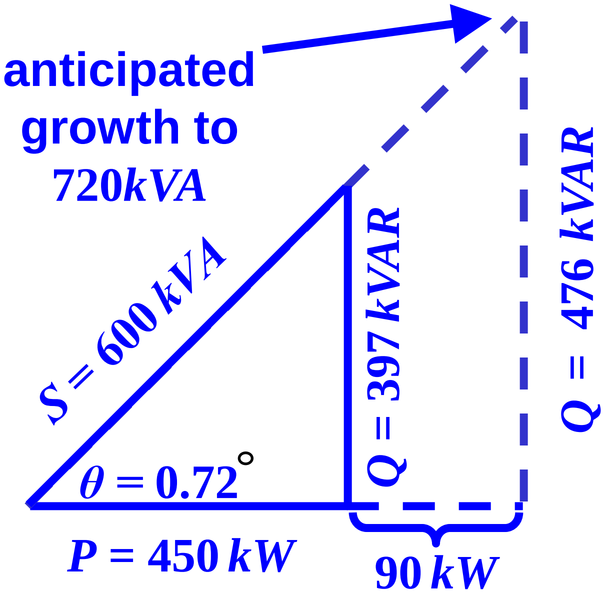
$$\theta = cos^{-1}\left(0.9\right) = 0.45\,radians$$

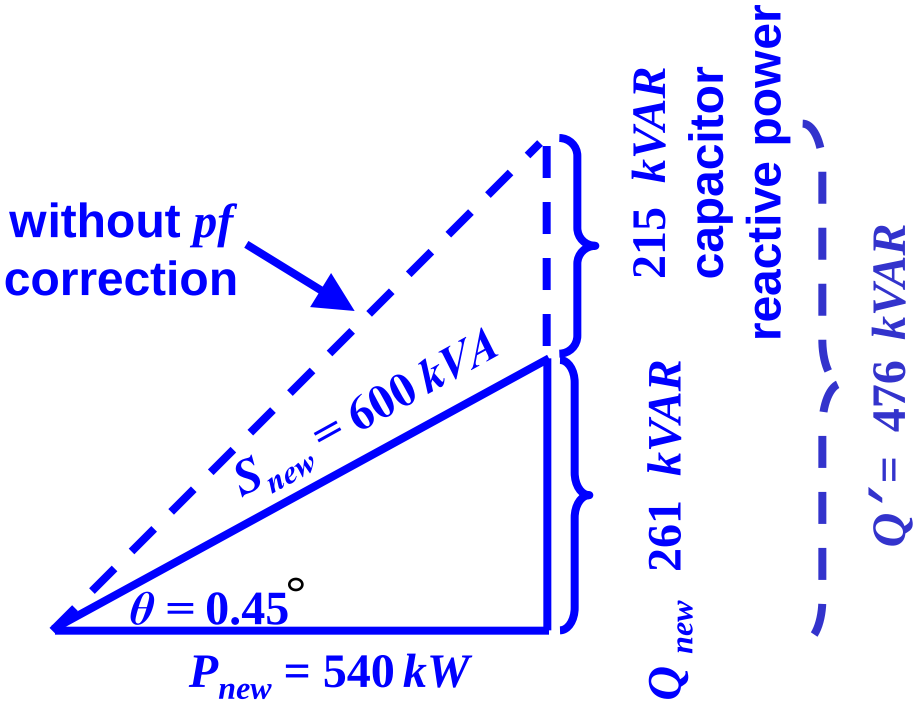
$$Q_{new} = 600\left(0.435\right) = 261\,kVAr$$

- ❑ The difference between $Q = 476\,kVAr$ and $Q_{new} = 261\,kVAr$ can be compensated by capacitors

$$Q_c = 476 - 261 = 215\,kVAr$$

EXAMPLE: POWER FACTOR CORRECTION





before correction

with 215-*kVAR* correction

EXAMPLE: POWER FACTOR CORRECTION

- ❑ We can determine the capacitance of the pf correcting capacitors

$$Q_c = V_c I_c = V_c \left(\omega C V_c \right)$$

$$C = \frac{Q_c}{\omega V_c^2}$$

- ❑ If we assume that the input voltage to the capacitors is at $12\,kV$, then

$$C = \frac{215\,kVAr}{(377)(12)^2 (kV)^2} = (3.96)\,10^{-3}\,F$$

DISTRIBUTION SYSTEM CAPACITORS FOR pf CORRECTION

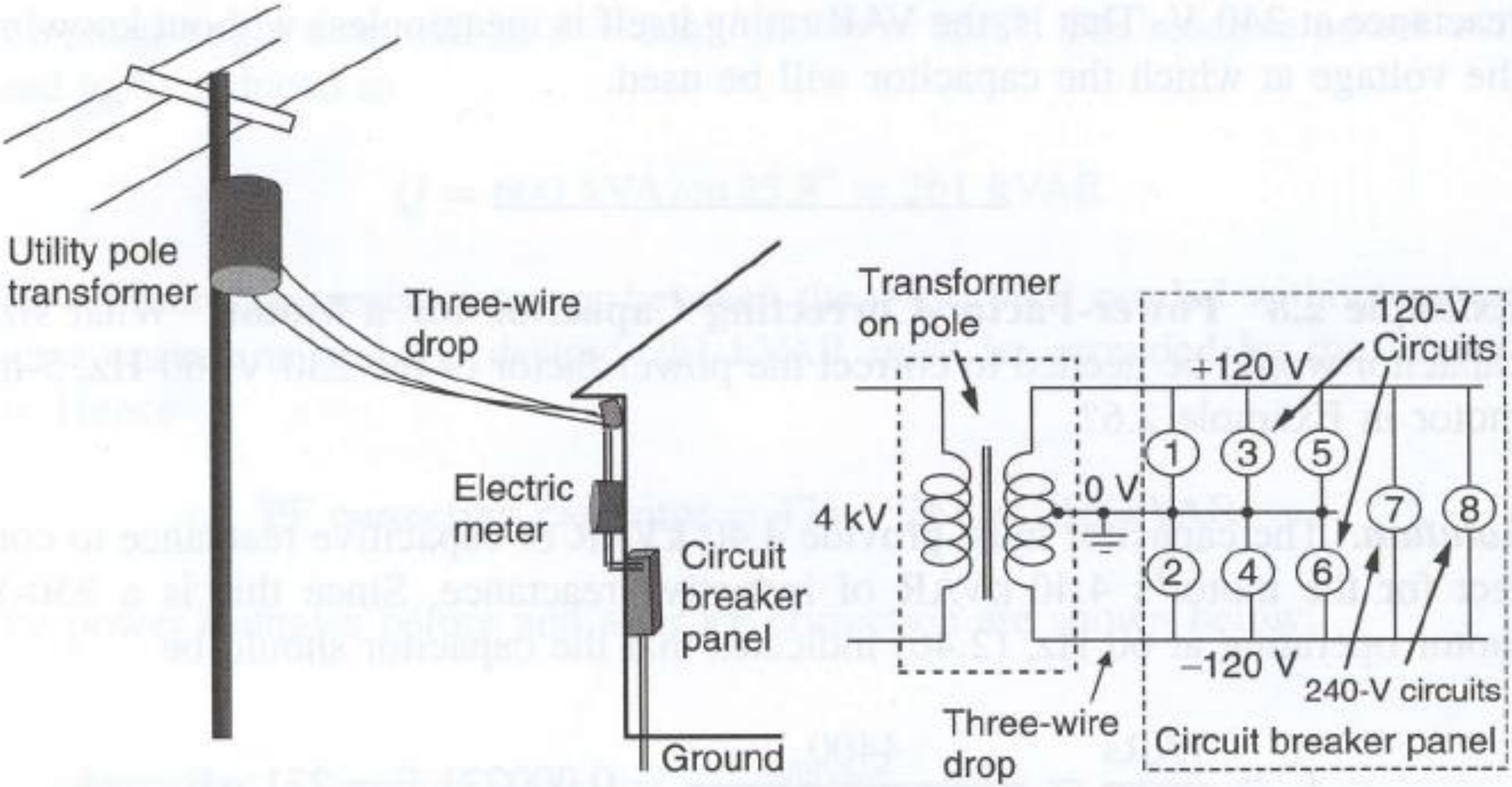
THE RESIDENTIAL ELECTRICITY SUPPLY

- In the U.S., residential service is typically provided from a $4.16\ kV$ feeder line through a step-down transformer to the $120/240\ V$ household voltage
  - all outlets provide $120\ V$
  - some outlets provide $240\ V$ electricity (air conditioning, heavier duty appliances)

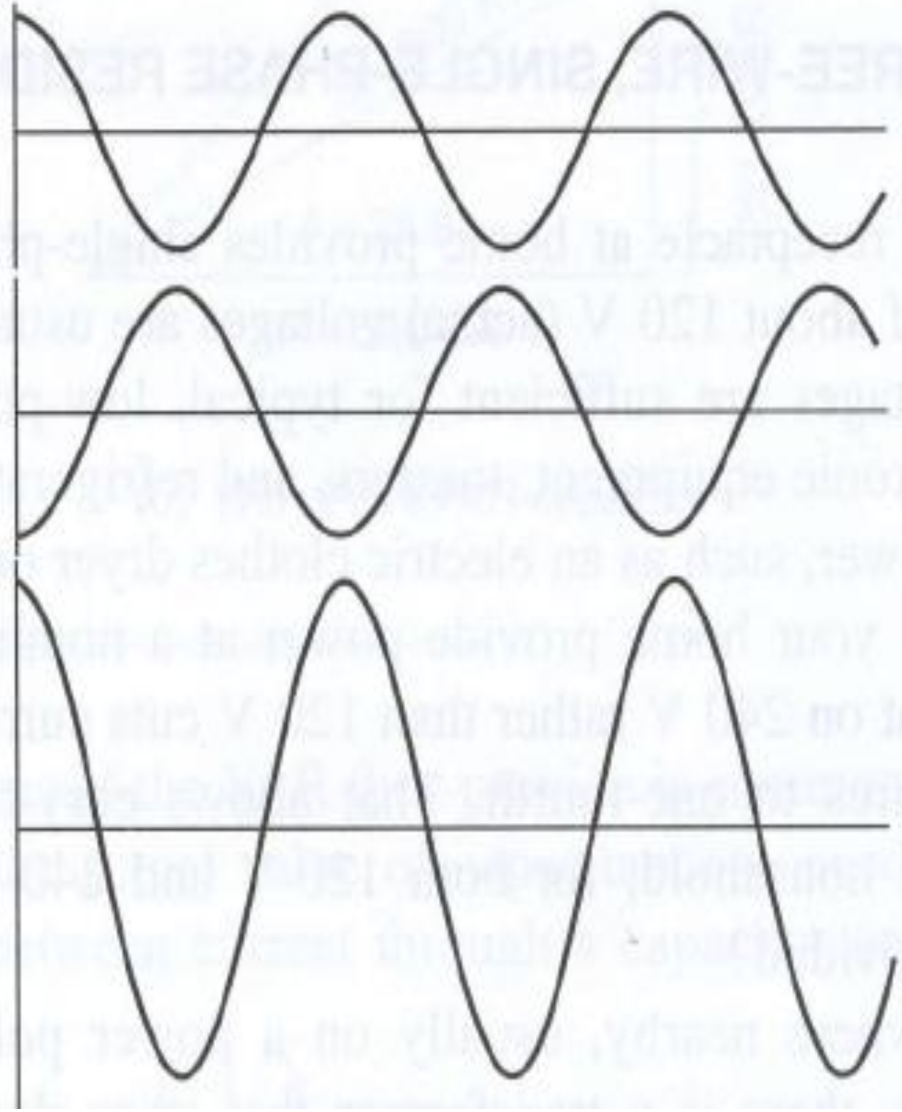
THE RESIDENTIAL ELECTRICITY SUPPLY

- The provision of $240\,V$ service is done by
  - grounding the center tap of the secondary side of the transformer
  - using the other two ends of the windings at the $\pm 120\,V$ supply to obtain the $240\,V$ potential

THE RESIDENTIAL ELECTRICITY SUPPLY



THE RESIDENTIAL ELECTRICITY SUPPLY



$$v_1 = 120\sqrt{2} \quad cos\left(377t\right)$$

$$v_2 = -120\sqrt{2} \quad cos\left(377t\right)$$

$$v_1 - v_2 = 240\sqrt{2} \quad cos\left(377t\right)$$

THE RESIDENTIAL ELECTRICITY SUPPLY

- ☐ **Analytically**

$$v_1(t) = 120\sqrt{2}\ \cos 377t$$

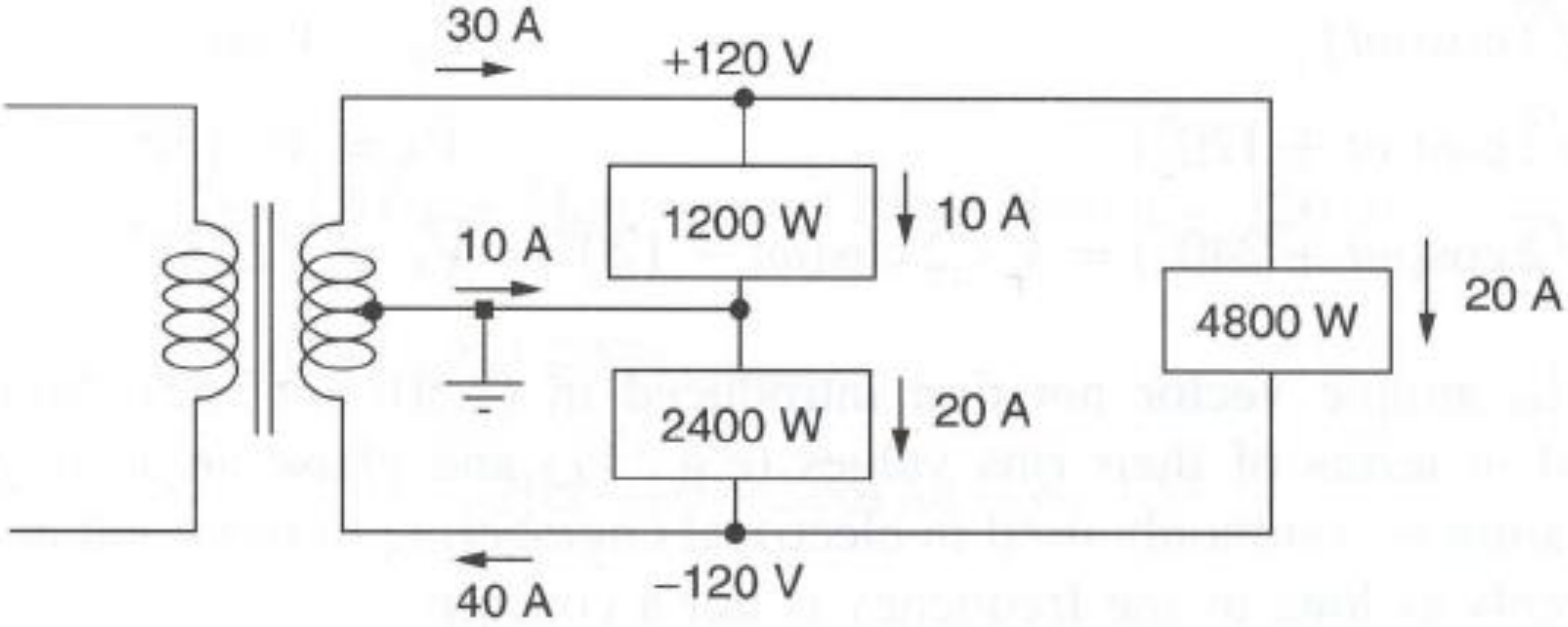
$$v_2(t) = 120\sqrt{2}\ \cos(377t + \pi)$$

$$= -120\sqrt{2}\ \cos 377t$$

and therefore

$$v_1(t) - v_2(t) = 240\sqrt{2}\ \cos 377t$$

RESIDENTIAL LOAD EXAMPLE



RESIDENTIAL LOAD EXAMPLE

- ❑ We consider the three loads served by a three-wire $120 / 240\,V$ system with

$$1,200\,W \text{ at } 120\,V \text{ on phase } A,\;\; p\,f = 1.0$$

$$2,400\,W \text{ at } 120\,V \text{ on phase } B,\;\; p\,f = 1.0$$

$$4,800\,W \text{ at } 240\,V,\;\; p\,f = 1.0$$

- ❑ We wish to compute the currents in the wires
- ❑ We start with the relationship

$$P = V\,I\,\cos\theta = V\,I$$

RESIDENTIAL LOAD EXAMPLE

- ☐ **For the $4{,}800\ W$ load**

$$I_{4,800} = \frac{4,800}{240} = 20\ A$$

- ☐ **For the $2{,}400\ W$ load**

$$I_{2,400} = \frac{2,400}{120} = 20\ A$$

- ☐ **For the $1{,}200\ W$ load**

$$I_{1,200} = \frac{1,200}{120} = 10\ A$$

RESIDENTIAL LOAD EXAMPLE

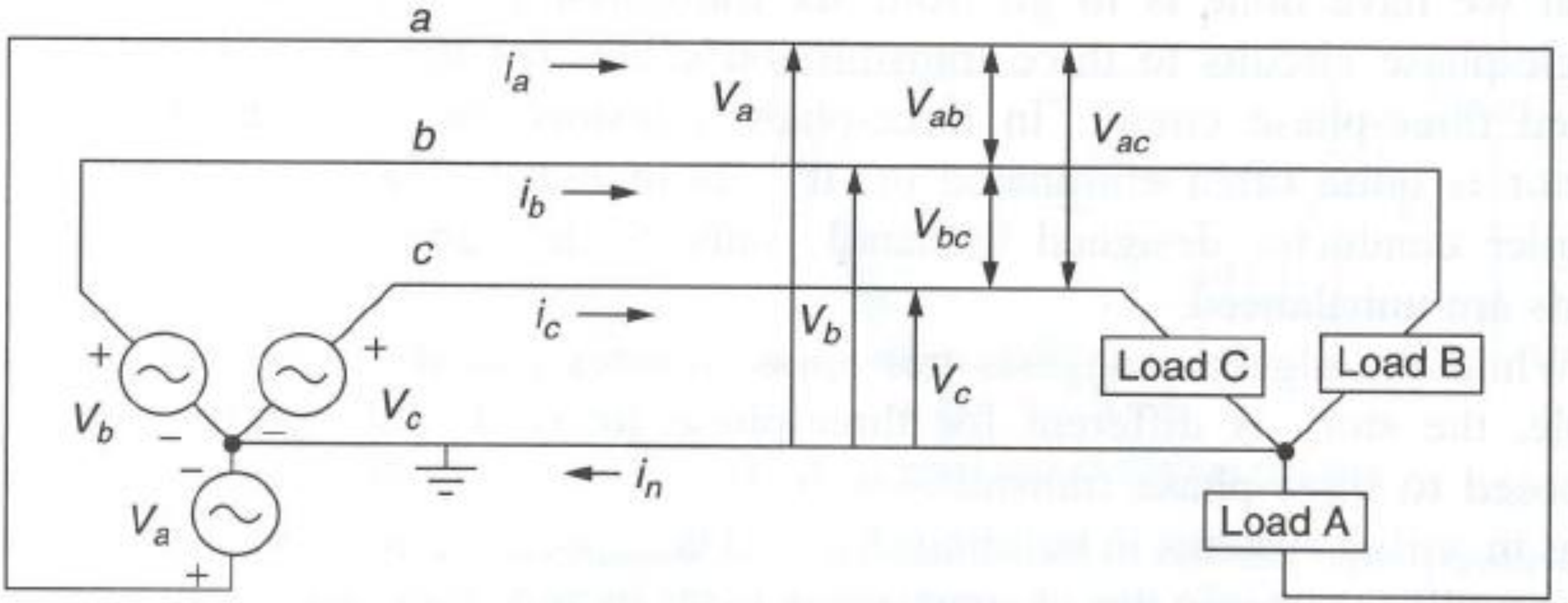
- ☐ **Note that *KCL* induces a current of 10 *A* *in the* neutral leg and therefore the unbalanced load creates a nonzero current in the neutral**

- ☐ **This case differs from the typical balanced conditions we encounter in which each *hot* leg has current of the same magnitude and the neutral current vanishes**

THREE – PHASE AC NETWORKS

- ☐ **Today's systems use the three – phase (3ϕ) generators to produce electricity and 3ϕ transmission lines to "transport" it to various parts of the network**
- ☐ **The interconnection of network elements into a 3ϕ network is done typically using either the (Δ) delta or wye (Y) configuration**
- ☐ **We examine a Y-connected 3ϕ generator to a 3ϕ load**

THREE – PHASE AC NETWORKS



THREE – PHASE AC NETWORKS

- ☐ The phase voltages are measured with respect to the neutral

$$v_a(t) = V\sqrt{2}\cos\omega t \quad \leftrightarrow \quad \vec{V}_a = Ve^{j0}$$

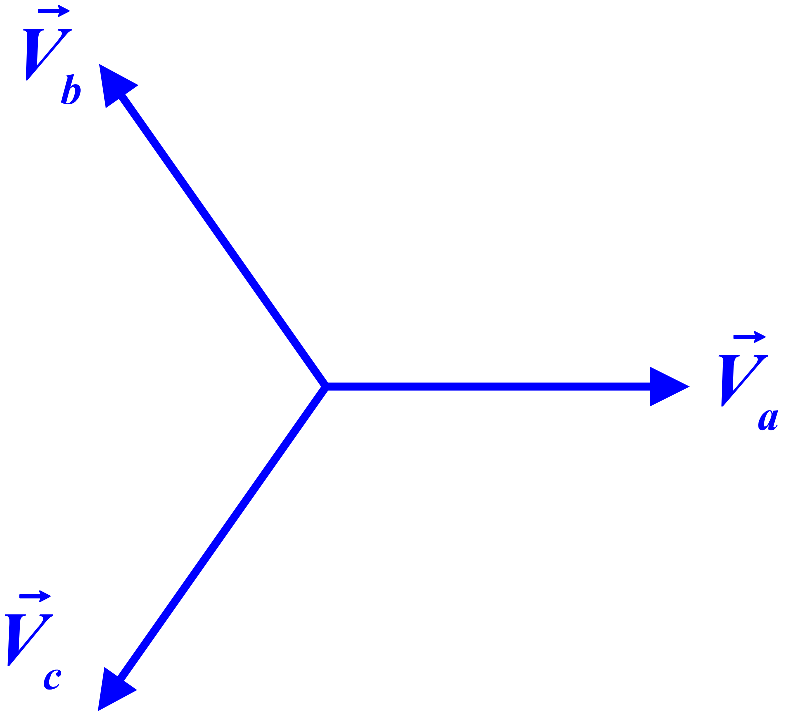
$$v_b(t) = V\sqrt{2}\cos\left(\omega t + \frac{2\pi}{3}\right) \quad \leftrightarrow \quad \vec{V}_b = Ve^{j\frac{2\pi}{3}}$$

$$v_c(t) = V\sqrt{2}\cos\left(\omega t - \frac{2\pi}{3}\right) \quad \leftrightarrow \quad \vec{V}_c = Ve^{-j\frac{2\pi}{3}} ,$$

with the entities on the right representing the phasor notation for the voltages

THREE – PHASE AC NETWORKS

- ☐ Note that the voltages are equal in magnitude and exactly $\pm \frac{2\pi}{3}$ from another (*balanced voltages*)



THREE – PHASE AC NETWORKS

- Consequently,

$$\vec{V}_a + \vec{V}_b + \vec{V}_c = 0$$

- The voltage between two-phases are typically called line voltages; for example the line *a* to the line *b* voltage is

$$v_{ab}\left(t\right) = v_{a0}\left(t\right) + v_{0b}\left(t\right) = v_{a0}\left(t\right) - v_{b0}\left(t\right)$$

and so

$$v_{ab}\left(t\right) = V\sqrt{2}\, cos\,\omega t - V\sqrt{2}\, cos\left(\omega t + \frac{2\pi}{3}\right)$$

THREE – PHASE AC NETWORKS

- ☐ Now, for a balanced network, the phase voltage *r.m.s.* values are equal

$$V_a = V_b = V_c = V_p \longleftarrow \quad \textit{r.m.s.} \text{ phase voltage}$$

- ☐ Therefore

$$v_{ab}(t) = V_p \sqrt{2} \cos \omega t - V_p \sqrt{2} \cos\left(\omega t + \frac{2\pi}{3} \right)$$

- ☐ We make use of the identity

$$\cos\phi - \cos\xi = -2 \sin\left[\frac{1}{2}(\phi + \xi) \right] \sin\left[\frac{1}{2}(\phi - \xi) \right]$$

THREE – PHASE AC NETWORKS

- ☐ So we obtain

$$v_{ab}\left(t\right) = V_p\sqrt{2}\cdot\left(-2\right) sin\left(\omega t + \frac{\pi}{3}\right)\cdot sin\left(-\frac{\pi}{3}\right)$$

$$= V_p\sqrt{2}\cdot 2 \ \ sin\frac{\pi}{3}\cdot sin\left(\omega t + \frac{\pi}{3}\right)$$

$$= \underbrace{\sqrt{3}\,V_p}_{V_\ell}\sqrt{2} \ \ sin\left(\omega t + \frac{\pi}{3}\right)$$

$$= V_\ell\sqrt{2} \ \ sin\left(\omega t + \frac{\pi}{3}\right)$$

THREE – PHASE AC NETWORKS

- ☐ The relationship of importance for the *r.m.s.* value of line-to-line voltage V_ℓ relative to that of the phase voltage V_p is

$$V_\ell = \sqrt{3}\, V_p$$

- ☐ Examples of typical values

service type	V_ℓ	V_p
buildings	202 V	120 V
commercial	480 V	277 V
residential	416 V	240 V

THREE – PHASE AC NETWORKS

- ☐ Each phase has apparent power

$$S_{\phi} = I_p V_p$$

and so the 3ϕ system has apparent power

$$\begin{aligned} S_{3\phi} &= 3 I_p V_p \\ &= \sqrt{3}\, I_p \sqrt{3}\, V_p \\ &= \sqrt{3}\, I_p V_{\ell} \end{aligned}$$

- ☐ Therefore,

$$P_{3\phi} = S_{3\phi} \cos\theta$$

$$Q_{3\phi} = S_{3\phi} \sin\theta$$

THREE – PHASE AC NETWORKS

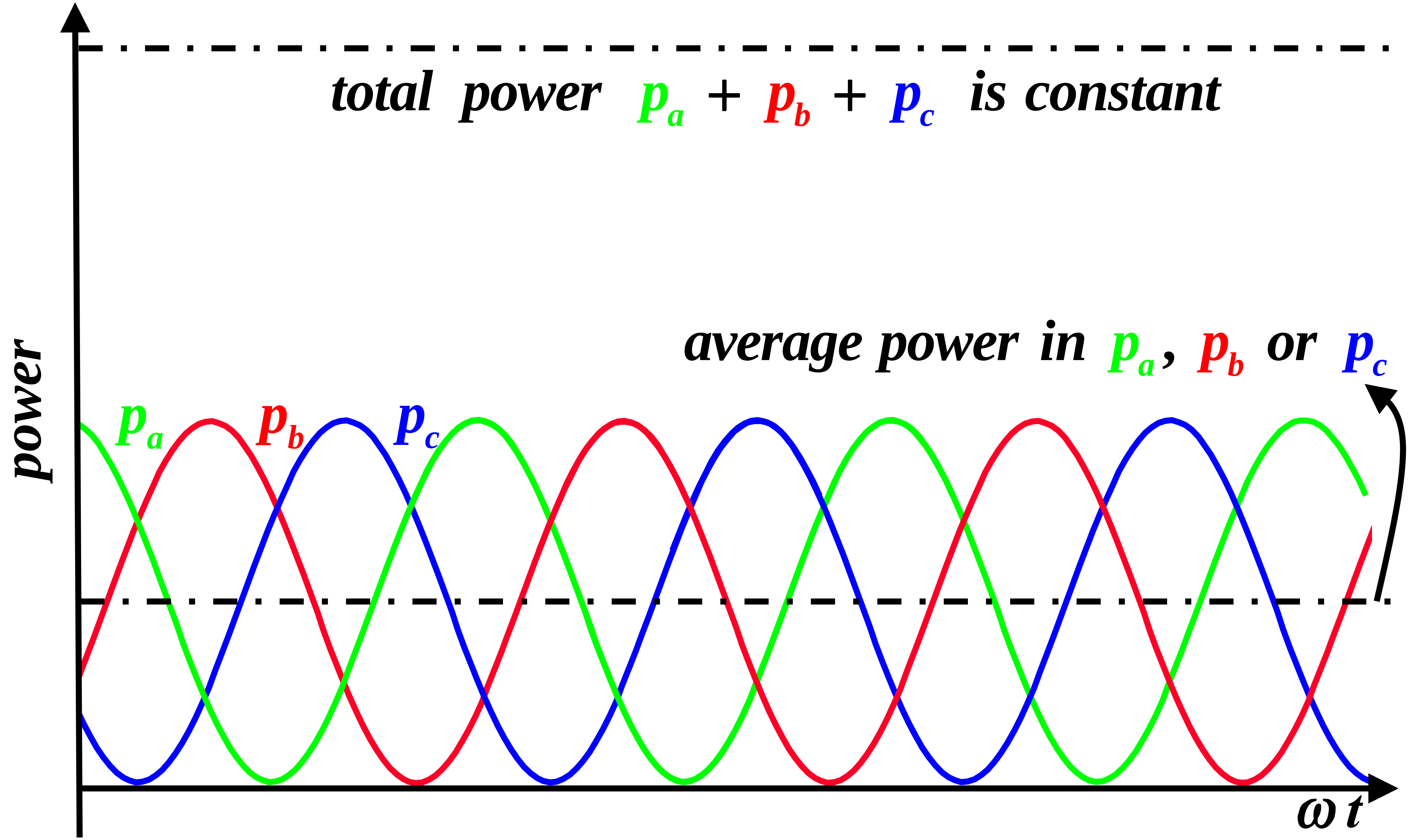
with θ being the phase angle between phase current and voltage and is the same for each phase under balanced conditions

- ☐ In fact, we can show that

$$p_a(t) + p_b(t) + p_c(t) = 3P_\phi$$

and is constant and such a smooth constant level of power constitutes a key advantage of 3ϕ systems in contrast to 1ϕ where $p(t)$ is sinusoidal

THREE – PHASE AC NETWORKS



EXAMPLE: 3ϕ NETWORK pf CORRECTION

- ☐ The 1ϕ motors in a small enterprise are supplied using a 3ϕ $208\,V$ transformer
- ☐ The real power demand is $80\,kW$ with a $pf = 0.5$ and incurs losses of $4\,kW$
- ☐ We compute $S_{3\phi}$ using

$$P_{3\phi} = \sqrt{3}V_{\ell}\,I_{p}\,cos\theta = S_{3\phi}\,0.5 = 80\,kW$$

so that

$$S_{3\phi} = 160\,kVA$$

EXAMPLE: 3ϕ NETWORK $p\,f$ CORRECTION

- ☐ We also evaluate

$$I_p = \frac{S_{3\phi}}{\sqrt{3}\,V_\ell} = \frac{160}{\sqrt{3}\;208} = .444\;kA$$

- ☐ Next consider a pf correction to 0.9 and so

$$S'_{3\phi} = \frac{80}{0.9} = 88.9\,kVA \ll 160\,kVA$$

EXAMPLE: 3ϕ NETWORK pf CORRECTION

- ☐ Also

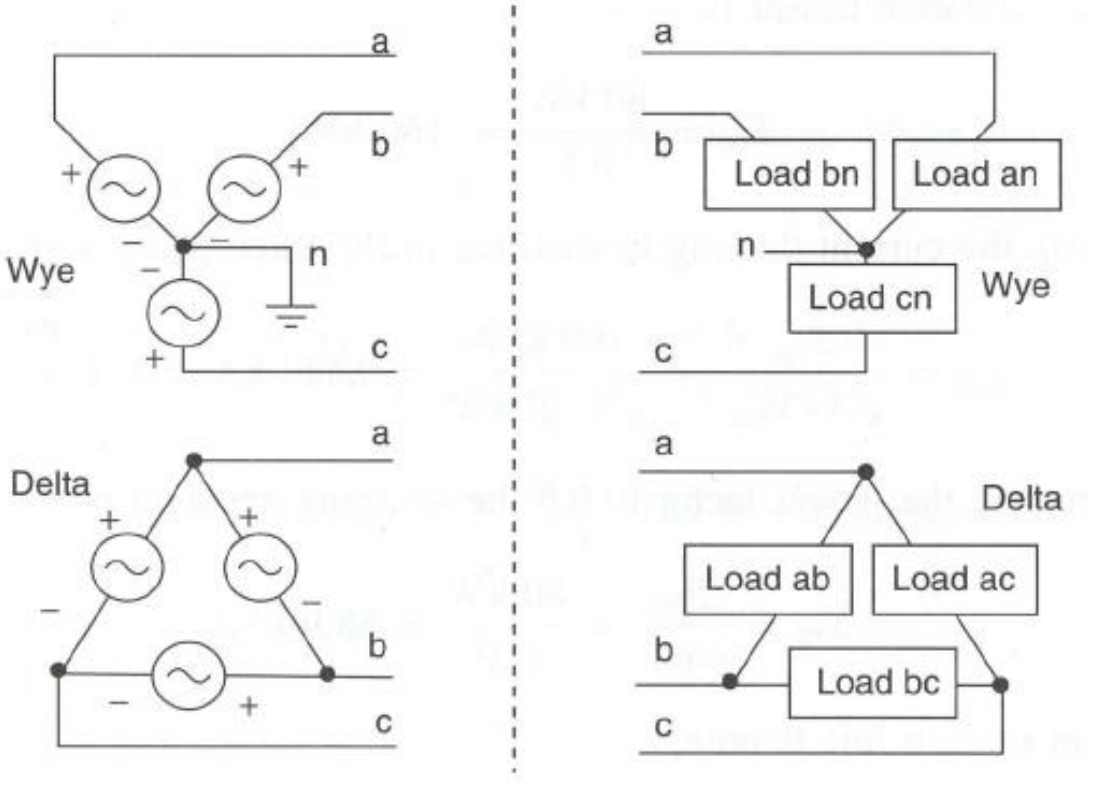
$$I'_p = \frac{88.9}{\sqrt{3}\,208} = .247\ kA$$

- ☐ We also evaluate the losses under corrected *pf*

$$R\left(I'_p\right)^2 = \frac{4}{\left(.444\right)^2}\left(.247\right)^2 = 1.24\ kW$$

THE 3ϕ DELTA CONNECTION

- The other way to connect 3ϕ elements in the Δ connection which has no neutral line



(a) Source (b) Load

THE 3ϕ DELTA CONNECTION

- ☐ **The comparison of the key characteristics of the two connection schemes is summarized by the table**

variable	$Y - connection$	$\Delta - connection$
r.m.s. current	$I_{\ell} = I_p$	$I_{\ell} = \sqrt{3}\, I_p$
r.m.s. voltage	$V_{\ell} = \sqrt{3}\, V_p$	$V_{\ell} = V_p$
3ϕ *power*	$P_{3\phi} = \sqrt{3}\, V_p I_p \cos\theta$	$P_{3\phi} = \sqrt{3}\, V_{\ell} I_{\ell} \cos\theta$